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One-dimensional Anderson model with dichotomic correlation

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Abstract. A one-dimensional diagonal tight binding electronic system with dichotomic correlated disorder is investigated. The correlation of random potential exponentially decays with distance and also with the dichotomic correlation parameter λ. Using a appropriate approximation, an analytical transmission coefficient expression is obtained. The obtained analytical expression is then tested against the result of the direct numerical computation of the average transmission coefficient $\langle T \rangle$ for the Anderson model, by changing the system parameters. In the thermodynamic limit the transmission coefficient relation indicates the absence of localization-delocalization transition, which is entirely consistent with numerical predictions.

PACS. 05.60.Gg Quantum transport – 72.15.Rn Localization effects (Anderson or weak localization) – 72.20.Ee Mobility edges; hopping transport – 64.60.Cn Order-disorder transformations; statistical mechanics of model systems

1 Introduction

According to Bloch's theorem, electronic states in perfectly ordered crystal are extended, which implies that the probability of finding an electron is the same over the entire system. Such extended states are an indication of metallic behavior. It has been more than four decades since Anderson pointed out that electron states become localized when disorder is introduced in the crystal, and the system can undergo a metal-insulator transition (MIT) [1,2]. It has also been known for almost forty years from scaling theory that in standard one-dimensional (1D) and two-dimensional (2D) disorder models all states are localized for any amount of disorder [3]. The phenomena of Anderson localization of electrons and the disorder-induced MIT have been studied since the introduction of the model [4,5]. While it may seem that after forty years of scrutiny nothing more could be said about the localization properties of 1D and 2D models, in fact their localization properties have recently attracted a great deal of attention. This renewed interest is due to mostly the correlation introduced in the disordered potential, which often causes an unexpected phenomenon: most intriguing is the breakdown of the Anderson localization in 1D systems. Breakdown of Anderson criterion of localization has been predicted for the first time for a random dimer model [6,7], wherein a short-range correlation has been introduced by a binary distribution. For a recent experiment using binary disorder see the reference [8]. A number of important studies on short range correlated disorder have also recently been worked out [9,10]. This unexpected result has motivated further studies into the nature of 1D systems with a long-range correlated disorder [11–14]. Especially in the system in which the potential sequences $\{\varepsilon_i\}$ has a power-law spectral density of the form $S(k) \propto k^{-p}$ [15,16] is particularly important. Here, k is the wave number of the wavelength of the undulation on the random parameter landscape. For exponent p is greater than 2.0, there is a finite range of energy eigenvalues with extended eigenstates [17–21]. This result indicates the presence of MIT contradicting the conclusion of the single parameter scaling theory. For exponent $p \leq 2$, although there are no extended eigenstates, the scaling behavior corresponding to this case has also opened up an interesting research field due to the interpretation of single parameter scaling theory (SPST) [22–30].

The most common element presented in a number of works references [23–25] is that they all have a power-law spectral density which can be defined by the exponent $p = 2$ at large k. This particular value of the exponent is important since $p = 2$ is the localization-delocalization critical point for the potential landscape of the long-range correlated disorder of references [17–21]. The author of reference [25] has studied the scaling properties of a particular form of correlated disorder that is associated with spin glass chains, moreover, has discovered that the scaling function shows a crossover near the band edge. The correlated random Markovian energy sequences have been analyzed in reference [26], and the authors observed that in some relevant situations the behavior of the Lyapunov exponent is different from the inverse of the correlation length.

The authors of reference [24] have shown that the introduction of dichotomic correlation in a random potential sequence, which is also going to be our focus in the scope

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of this paper, leads to an additional parameter governing the validity of SPST. In their work they have calculated the approximate average Lyapunov exponent by considering three spectral regions with different transport properties. In the context of their work, this approximation means neglect of commutators between one-step transfer matrices at different sites. The relevance of the obtained average Lyapunov exponent and the new scaling parameter were tested by comparing them with the direct numerical calculations. We think that the statement of their approximation is very understandable and clear; however its use and justification in the derivation of the Lyapunov exponent is quite confusing and ambiguous. Although this current paper is mainly inspired by references [23,24], in our calculations we are going to use a different approximation with the hope that it might be more understandable and also more easily repeated in the future.

In the present paper, we focus upon correlation induced changes in the transmission coefficient relation. To do so, we have considered a tight-binding model with random correlated potential described by a zero-mean dichotomic process. The potential sequence in dichotomic processes is defined as $\varepsilon_n = V\chi_n$; here V is the amplitude of the potential and χ_n is the dichotomic processes assuming only the values of ± 1 with Poisson probability. Therefore, the dichotomic processes is very similar to the well known binary processes. However, to build a binary random potential sequence two different potential values ε_A and ε_B are assigned at random to each lattice site with probability P and $1-P$ respectively. The utility of preferring a dichotomic process is that it allows for an approximate analytical treatment of the transmission coefficient. In the following section, we will describe the theoretical model which is going to be used in this work together with some properties of the dichotomic process. In the third section, an average analytical transmission coefficient will be derived. By comparing the analytical and numerical calculations, the relevance of the analytical expressions are also going to be tested in the same section. Further, making use of this analytical relation, we are going to try to evaluate and test the prediction found in references [17–21] for $p = 2$ analytically. In the last section, we are going to investigate the transmission coefficient distribution function and also the transmission coefficient probability density function. In the same section, the cumulants of the scaling variable $\ln T$ are also going to be calculated numerically in order to obtain a possible analytical relation between them.

2 Theoretical model

2.1 Conductance and dichotomic process

As a model system, we considered noninteracting electrons in a one-dimensional dichotomic correlated disordered system within a tight binding approximation. For a discrete lattice chain, the Schrödinger equation of the model is expressed as

$$V\chi_i\psi_i + t(\psi_{i+1} + \psi_{i-1}) = E\psi_i \tag{1}$$

where ψ_i is the amplitude of the wave function at the ith site of the lattice, V describes the strength of the random potential and χ_i is the dichotomic random variable which takes the values of 1 or -1. The overlap integral parameter or the hopping energy t is going to be set to unity in the following and, thus, a sequence of correlated disorder potential is going to be produced by the dichotomic process. The localization length ξ of a state with energy E is given [31] as

$$\frac{1}{\xi_L} = -\frac{1}{2L}\langle \ln T \rangle \tag{2}$$

where L is the length of the chain which can be set to an integer N if the lattice spacing is set to unity, $\langle \dots \rangle$ denoting the average over the possible realization of the system. Here, T is the transmission coefficient which is related to the eigenvalues of the matrix Q defined as

$$Q = T_M^T T_M \tag{3}$$

where T_M is the transfer matrix. For the discrete Hamiltonian lattice model, the propagation of the excitation along the system can be expressed in the following form:

$$\begin{pmatrix} \psi_{i+1} \\ \psi_i \end{pmatrix} = \begin{pmatrix} E-\varepsilon_i & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi_i \\ \psi_{i-1} \end{pmatrix} \tag{4}$$

where the one-step transfer matrix T_i is defined as

$$\begin{pmatrix} E-\varepsilon_i & -1 \\ 1 & 0 \end{pmatrix}. \tag{5}$$

The transfer matrix T_M describing the evolution of the initial state vector across N sites can be readily expressed as the product of one-step matrices if the distance between adjacent sites is set to unity, that is

$$T_M = \prod_i^N T_i. \tag{6}$$

The eigenvalues of Q are real positive numbers coming in inverse pairs. Expressing the eigenvalues as $q_{i,1} = e^{\nu_i}$ and $q_{i,2} = e^{-\nu_i}$, the transmission eigenvalues can be given by

$$T = \frac{2}{1+\cosh(\nu_i)}. \tag{7}$$

The conductance of a 1D transport is defined by the Landauer formula as $g = T$ in the unit of $2e^2/h$ [32–34]; here the factor of 2 is due to the two possible spin states of an electron.

2.2 Dichotomic correlated potential sequence

In this paper we want to consider the special effects of spatially correlated potential sequence on the transport properties of the system. For this reason we will use a dichotomic rule to generate the random potential sequence. The dichotomic or random telegraph process can be summarized briefly in the following manner.

Let consider a random process that has the properties:

i) the dichotomic variables χ_n have only the ± 1 values;
ii) the probability of the initial dichotomic variable χ_0 is the same, that is, $P(\chi_0 = 1) = P(\chi_0 = -1) = 1/2$;
iii) defining $\chi_n = \chi_0(-1)^{N_n}$, here N_n denotes a Poisson process and n is the nth lattice site, a dichotomic process can be created. The probability of N_n is defined by the Poissonian process as

$$P(N_n = m) = \frac{(\alpha n)^m e^{-\alpha n}}{m!}. \tag{8}$$

It is well-known that this process has a zero mean, that is $\langle \chi_n \rangle = 0$, and so the two points correlation function corresponding to this process can also be expressed [35] as

$$\langle \chi_n \chi_{n+\ell} \rangle = e^{-2\alpha \ell} \tag{9}$$

where ℓ is the distance between the sites n and $n + \ell$. The probability of having the values of 1 or -1 for each lattice site is given as $P(\chi_n = 1) = e^{-\alpha}\cosh(\alpha)$ and $P(\chi_n = -1) = e^{-\alpha}\sinh(\alpha)$, respectively if the initial dichotomic variable is set to 1. Thus, a lattice consisting of N site has the number of site with the value of 1, $N_+ = NP(\chi_n = 1)$, and the number of site with a -1 value, $N_- = NP(\chi_n = -1)$; here apparently N is equal to $N_+ + N_-$. We think it is important to mention that the dichotomic process described above is same as the dichotomic process used in reference [24] if the definition of the inverse of correlation radius $\frac{1}{r_c} = -\lim_{l->\infty} \frac{1}{l} \ln\langle \xi_{n+l}\xi_n \rangle$ (or the correlation length for the random potential Ref. [26]) is considered. Thus, $\frac{1}{r_c}$ is equal to 2α in the formulation of this paper. And also, the assumed value remains constant within the region of the random segment l_c is equal to $\coth \alpha$

The other important quantity in the study of correlated disorder is the power spectral density $S(k)$ which is defined as the Fourier transform of the two points correlation function. Thus, evaluation of the Fourier transform of equation (9) leads to

$$S(k) = \frac{\alpha\sqrt{\frac{\pi}{2}}}{k^2 + 4\alpha^2}. \tag{10}$$

The form of the above spectral density is equivalent to the $p = 2$ case studied in references [17–21] for $\alpha \ll k$. This observation is a good indication of the absence of MIT in the case of dichotomic correlated disorder if the results of the cited references are considered. Thus, a further investigation into the existence of Metal-Insulator transition in the presence of dichotomic correlated disorder in the Anderson model turns out to not be an interesting point. However, we think that studying analytically the transmission properties of the system of a dichotomic correlated Anderson model is certainly a valuable point to consider. Therefore, in the following section, we are going to mainly focus upon to obtaining an approximate analytical relation for the transmission coefficient of this system.

3 Transmission coefficient relation for the dichotomic process

In this section, we investigate analytically the transport properties of electronic state moving in a lattice endowed by a dichotomic correlated random potential sequence. The transport nature of the system can be best described by the transmission coefficient or equivalently by the conductivity of the system introduced in the above section. For most potential sequences, a numerical evaluation of these quantities are quite simple in contrast to an analytical treatment. This is due to the random nature of the potential sequences which are inserted into the one-step transfer matrices T_i describing the change of the state in one discreet step. In the case of a dichotomic process, however, the potential sequence takes only the values which are either 1 or -1. Thus, the transfer matrix T_M describing the evaluation of the initial state across N sites turns out to be the product of the two distinct one-step matrices. This last point is the most important property of the dichotomic process in a possible analytical treatment of the transmission coefficient and will be employed wherever necessary. Depending on the values of the dichotomic process χ_i, the one-step transfer matrices $T^{\pm}$ become

$$\begin{pmatrix} E \pm V & -1 \\ 1 & 0 \end{pmatrix}. \tag{11}$$

An expression relating T^+ to T^- can also be written quite readily as $T^- = T^+ + A$, here A is a 2×2 matrix which can be written as

$$\begin{pmatrix} -2V & 0 \\ 0 & 0 \end{pmatrix}. \tag{12}$$

Thus, the product of one-step matrices turns out to take the following possible form

$$T_M = T_1^+ T_2^+ \ldots T_{N_1}^+ T_{N_1+1}^- T_{N_1+2}^+ \ldots \tag{13}$$

Writing T^- in terms of T^+, the last equation can be rearranged approximately as

$$T_M = (T^+)^N + N_-(T^+)^{N_1} A (T^+)^{N-N_1-1} + \frac{N_-!}{2!(N_- - 2)!}(T^+)^{N_1} A (T^+)^{N_2} A (T^+)^{N-N_1-N_2-2} \ldots \tag{14}$$

In equation (14), the approximation means neglecting the commutators between A and T^{N_i} wherever it is necessary as already used in reference [24]. Here N_- denotes the number of sites having -1 potential value, $N_1, N_2, \ldots, N_i$ are the number corresponding to the potential landscape having the potential values of 1 without assuming any -1 values. In what follows, we find it more convenient to write T^+ in diagonal form as $D\Lambda D^{-1}$, here the matrices Λ, D

and D^{-1} are equal to

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}, \tag{15}$$

$$D = \begin{pmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{pmatrix}, \tag{16}$$

$$D^{-1} = \begin{pmatrix} -m & \frac{m\lambda_2}{2} \\ m & -\frac{m\lambda_1}{2} \end{pmatrix}. \tag{17}$$

In the above matrices the quantities $\lambda_1 = (a - \sqrt{a^2-4})/2$, $\lambda_2 = (a + \sqrt{a^2-4})/2$ and $m = 1/(\sqrt{a^2-4})$. Here a is equal to $|E - V| \leq 2$. Defining $B = D^{-1}AD$, the transfer matrix T_M turns out to be

$$T_M = D\Big[\Lambda^N + 2VN_-\Lambda^{N_1}B\Lambda^{N-N_1-1} + \frac{N_-!}{2!(N_--2)} 4V^2\Lambda^{N_1}B\Lambda^{N_2}B\Lambda^{N-N_1-N_2-2} + \cdots\Big]D^{-1}. \tag{18}$$

In each term of the above T_M relation, there are matrix elements coming in the form of $(\lambda_1/\lambda_2)^{N_i}$. If the simple relation: λ_1 is equal to the complex conjugate of λ_2, $\lambda_1 = \lambda_2^*$, is considered, the following approximation can be used in the evaluation of the transfer matrix without omitting the basic elements of physics. For our analysis we think it is convenient to express the ratio λ_1/λ_2 in the following complex function form as

$$\lambda_1/\lambda_2 = e^{i\pi\phi} \tag{19}$$

where ϕ is equal to $\arctan(2a\sqrt{4-a^2}/(2a^2-4))$. Using the standard relation $\mid \lambda_1/\lambda_2 \mid = 1$, the averaging of the matrix elements over the possible values of N_i leads to $\langle(\lambda_1/\lambda_2)^{N_i}\rangle \simeq 0$ if The number N_i is random for each realization of the system. In other words, this approximation is very similar to the random phase approximation as already used in reference [2] in which the averaging is taken over the scattering angle. At this point we simply assume that the approximation is a appropriate one so long as the correlation between the -1 potential values is omitted. Thus making use of this approximation T_M can be expressed as

$$T_M = D\Big[\Lambda^N + 2VN_-C_1 + 4V^2\frac{N_-!}{2!(N_--2)!}C_2 + \cdots\Big]D^{-1}, \tag{20}$$

where C_1 and C_2 are equal to the following matrices

$$C_1 = \begin{pmatrix} -m\lambda_1^N & 0 \\ 0 & m\lambda_2^N \end{pmatrix}, \tag{21}$$

$$C_2 = \begin{pmatrix} m^2\lambda_1^N & 0 \\ 0 & m^2\lambda_2^N \end{pmatrix}. \tag{22}$$

T_M can also be expressed more conveniently as $T_M = DCD^{-1}$. Here the matrix C is equal to

$$C = \begin{pmatrix} (1+k_1)\lambda_1^N & 0 \\ 0 & (1+k_2)\lambda_2^N \end{pmatrix} \tag{23}$$

where k_1 and k_2 have the following expressions

$$k_1 = \sum_{j=1}^{N_-} \frac{N_-!}{j!(N_--j)!}(-2Vm)^j, \tag{24}$$

$$k_2 = \sum_{j=1}^{N_-} \frac{N_-!}{j!(N_--j)!}(2Vm)^j. \tag{25}$$

Evaluations of these sums lead to $k_1 = (1-2Vm)^{N_-} - 1$ and $k_2 = (1+2Vm)^{N_-} - 1$. These final results allow us to calculate the eigenvalues of the matrix $Q = T_M^T T_M$ quite readily. Evaluation of the eigenvalues leads to the following expression for their sum

$$q_1 + q_2 = \frac{2}{1-\frac{a^2}{4}}(1+4V^2m^2)^{N_-} \tag{26}$$

where, q_1 and q_2 are apparently the first and the second eigenvalues of the matrix Q. Making use of this final expression, a relation for the average transmission coefficient of the system $\langle T\rangle$ can be readily obtained as

$$\langle T\rangle = \frac{2}{1+\frac{4}{4-a^2}(1+4V^2m^2)^{N_-}}. \tag{27}$$

It is thus of interest to compare the analytical transmission coefficient relation with the numerical data. The numerical results calculated using equation (7) have been obtained by averaging over 10^3 realizations of the system. Figures 1 and 2 present the result of numerical data together with the corresponding analytical curves by setting the energy parameters $E = 0$ and $E = 0.5$ respectively. It is well-known that the random phase approximation in Anderson model for uncorrelated potential sequences at $E = 0$ breaks down [36–38] and see the reference [29]. However, in both figures one can see that the analytical relation provides a reasonably good approximation of the numerical data. Furthermore, for the values of disorder strength V less than 0.5 and larger than 1.5, the agreement between the analytical results and numerical calculations is almost perfect. In order to confirm a generic nature of applicability of the analytical result, we have also carried out numerical calculations for all possible energy values at small disorder strength. The results have been presented in Figure 3. The general trend of these curves seems to follow the case of uncorrelated disorder as expected. From the figure it is easy to see that the agreement between the analytical and numerical results is quite acceptable. Therefore it is relevant to claim that the analytical expression derived in this paper covers the general feature of transport properties of dichotomic disorder model for small disorder strength and small Poisson parameter α for all values of energy.

To reveal the utility of the analytical transmission coefficient relation, let us calculate its tendency in the thermodynamic limit, namely in the limit as N goes to infinity. Evaluation of this limit for infinitely small disorder strength leads to $\langle T\rangle = 0$. That means there is no extended state in the thermodynamic limit, which indicates

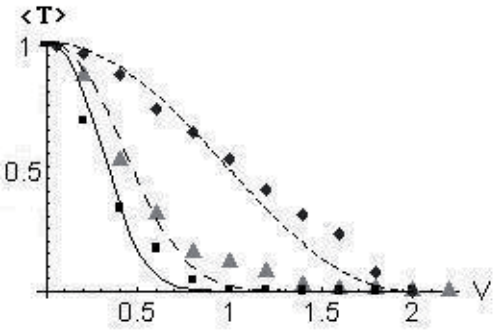

Fig. 1. Comparison of the analytical transmission coefficient relation with the numerical data for $E = 0$ at various disorder strength V. Numerical data shown by the diamond symbol is for $\alpha = 0.001$, by the triangle symbol is for $\alpha = 0.005$, and by the square symbol for $\alpha = 0.01$. The analytical result is plotted with the short dashed line for $\alpha = 0.001$, with the long dashed line for $\alpha = 0.005$, and with the continuous line for $\alpha = 0.01$. N is set to 1000 in all considerations.

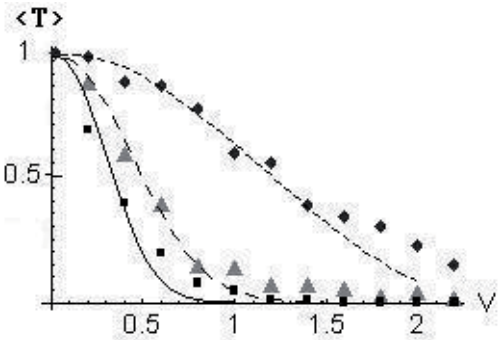

Fig. 2. Comparison of the analytical transmission coefficient relation with the numerical data for $E = 0.5$ at various disorder strength V. Numerical data shown by the diamond symbol is for $\alpha = 0.001$, by the triangle symbol is for $\alpha = 0.005$, and by the square symbol for $\alpha = 0.01$. The analytical result is plotted with the short dashed line for $\alpha = 0.001$, with the long dashed line for $\alpha = 0.005$, and with the continuous line for $\alpha = 0.01$. N is set to 1000 in all considerations.

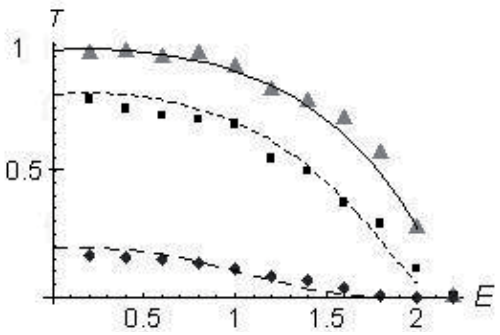

Fig. 3. Comparison of the analytical transmission coefficient relation with the numerical data for $V = 0.2$ at various energy levels E. Numerical data shown by the diamond symbol is for $\alpha = 0.1$, by the triangle symbol is for $\alpha = 0.001$, and by the square symbol for $\alpha = 0.01$. Analytical result is plotted with the short dashed line for $\alpha = 0.01$, with the long dashed line for $\alpha = 0.1$, and with the continuous line for $\alpha = 0.001$. N is set to 1000 in all considerations.

the absence of metal-insulator transition for the system. This totally expected result is important in that it is not only in agreement with the predictions of the known numerical studies but also with the prediction of scaling theory. In other words, the role played by the correlation of dichotomic process does not affect the main prediction of scaling theory. This conclusion is the prominent utility of the analytical transmission coefficient relation derived in this paper. As shown by the above analytic treatment, while all states in this model are localized, the scaling properties of the model are still interesting from the point of view of single parameter scaling theory [39–42]. Of course, this is no easy task, but it might be worth pursuing in the future.

In summary, in this paper 1D transport properties of dichotomic correlated disorder have been studied analytically, and an analytical transmission coefficient expression has been derived in terms of the relevant parameters of the model. This analytic expression has been tested against the result of direct numerical computation of the average transmission coefficient $\langle T \rangle$ for the Anderson model, by changing the system parameters. For all values of the energy eigenvalues, the agreement between the analytical expression and numerical data is perfect at small disorder strength and small Poisson parameter α. We think that the agreement shows the relevance of the approximation used in the evaluation of the analytical conductivity relation. We have approached the question concerning the importance of the analytical transmission expression by utilizing it to demonstrate the absence of localization-delocalization transition in the thermodynamic limits analytically.

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